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On convection velocities in turbulent shear flows

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The problem of defining an effective velocity of convection for turbulent fluctuations in a shear flow is considered, and the definitions adopted by various workers are discussed. An experiment in the shear layer of a circular jet has shown that the usual definitions, based on peaks of the space–time correlations of the fluctuations, yield convection velocities whose magnitudes depend on the value of space or time separation chosen. An alternative approach shows that, by considering the turbulent field as a superposition of harmonic travelling waves, a wave-number/velocity spectrum can be defined that lends itself to the definition of a wave-number-dependent convection velocity and an overall convection velocity, both of which have real physical significance. An experimental technique is described for obtaining the spectrum, and results are presented for one position in the shear layer of the jet.

1. Introduction

The idea of a velocity of convection of turbulent fluctuations stems originally from Taylor's work on grid turbulence. Taylor (1938) pointed out that, if the turbulence level were low, the time variations in the velocity u observed at a fixed point in the flow would be approximately the same as those due to the convection of an unchanging spatial pattern past the point with the mean flow velocity U, i.e. that $u(x,t) \doteqdot u(x-Ut, 0)$, where x and t represent distance measured downstream in the mean flow direction and time. This hypothesis has become known as Taylor's hypothesis, and Taylor (1938) showed experimentally that it was approximately true for the case of turbulence behind a grid in a wind tunnel.

From a consideration of the full Navier–Stokes equations, Lin (1952) has shown that Taylor's hypothesis is valid only if the turbulence level is low, viscous forces are negligible, and the mean shear is small. It is thus unreasonable to expect the hypothesis to apply throughout a boundary layer or in the mixing region of a jet. Nevertheless, the idea that slowly distorting eddies are convected downstream by the mean flow at a steady velocity (not necessarily equal to the mean velocity) is a useful one in the study of turbulent shear flows, and is particularly important in the study of aerodynamic noise and turbulence-induced structural vibrations. Various experiments have been conducted to determine effective convection velocities of turbulent fluctuations in shear flows, and such velocities have usually been defined in terms of the space–time covariance of velocity or pressure. Different workers have adopted their individual definitions according to the particular interest in the flow; only in the case of a frozen convected pattern, that of Taylor's model, is the choice of convection velocity unambiguous.

2. Theoretical discussion

Taylor's hypothesis may be regarded as an assumption that the fluctuating velocity, written in the form $u(x+U\tau_1, t+\tau_1)$, is independent of the value of τ_1. For a stationary and spatially homogeneous flow we can define the velocity covariance $R(\delta, \tau)$ as the time mean of the product $u(x,t)\,u(x+\delta, t+\tau)$, which by Taylor's hypothesis is equal to the mean value of

$$\overline{u(x,t)\,u(x+\delta+U\tau_1, t+\tau+\tau_1)} = R(\delta+U\tau_1, \tau+\tau_1).$$

Thus, according to Taylor's hypothesis, the covariance $R(\delta+U\tau_1, \tau+\tau_1)$ is also independent of the value of τ_1.

It is interesting to note that the same invariance of the covariance

$$R(\delta+U\tau_1, \tau+\tau_1)$$

can be derived from other simple assumptions that do not necessarily imply a frozen convected pattern. For instance, if we assume that the change in the turbulence pattern between the two measuring points is statistically independent of the instantaneous velocity fluctuation at the first point, i.e. if

$$\overline{\partial^n u(x,t)/\partial t^n\, \partial^m[u(x,t)-u(x+\delta, t+\tau)]/\partial t^m} = 0 \quad \text{for some value of } \delta/\tau, \text{ all } n, m, \tag{2.1}$$

then for $n = m = 0$ we immediately have

$$\overline{u^2(x,t)} = \overline{u(x,t)\,u(x+\delta, t+\tau)} \quad \text{for some } \delta/\tau, \tag{2.2}$$

or $R(\delta, \tau) = R(\delta+U\tau_1, \tau+\tau_1)$ as before. In an x-homogeneous flow it is easily shown that equation (2.2) can be satisfied only by the frozen convected pattern, and in a flow in which the turbulent intensity $\overline{u^2(x,t)}$ decreases with increasing x the condition cannot be satisfied at all. In the latter case, we might suppose instead that equation (2.1) holds for the velocities normalized by the r.m.s. intensities at the two points, when we obtain the result that the correlation coefficient $R(\delta+U\tau_1, \tau+\tau_1)/[\overline{u^2(x,t)}\;\overline{u^2(x+\delta,t)}]^{\frac{1}{2}}$ is independent of the value of τ_1. It is then easily shown that the velocity change between the two points can only be a constant factor reduction, the factor being the ratio of the r.m.s. intensities at the two points.*

Measurements of the covariance $R(\delta, \tau)$ or of the correlation coefficient in turbulent shear flows show marked differences from this invariance to a uniform translation of axes, as shown in figures 1 and 2. Figure 1 shows lines of constant covariance in Taylor's model in the (δ, τ)-plane, and the convection velocity U is given unambiguously by the constant slope $(d\delta/d\tau)_{R=\text{const}}$. Figure 2 is an exaggerated version of the type of pattern measured by Willmarth & Wooldridge (1962) for pressure fluctuations under a turbulent boundary layer. Here the slope of a line of constant covariance no longer has a constant value, but can take all values over a range of δ or τ. The concentration of appreciable correlation into a fairly narrow band in the diagram indicates, however, that convection still plays

* I am indebted to a referee for pointing out these alternative hypotheses leading to the invariance properties of the covariance or correlation coefficient.

an important part in the flow and suggests possible definitions of a convection velocity. The covariance is measured experimentally by making surveys either with a series of constant space separations δ and varying time delay τ, or vice versa. An obvious definition of convection velocity in the former case, for a particular separation, is the value of the ratio δ/τ which makes the covariance a maximum, i.e. the value δ_1/τ_c, where τ_c satisfies

$$\partial R(\delta_1, \tau)/\partial\tau = 0. \tag{2.3}$$

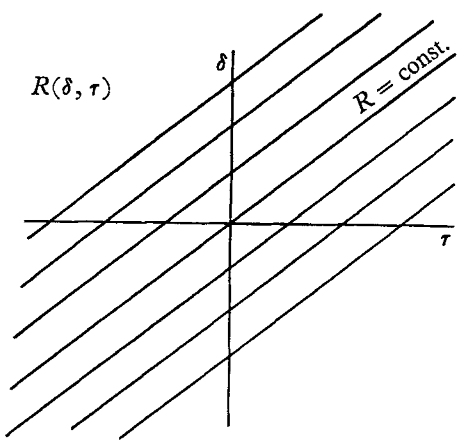

FIGURE 1. Space–time covariance in Taylor's model.

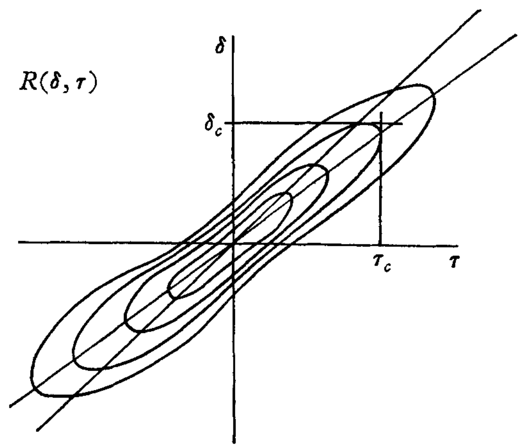

FIGURE 2. Practical space–time covariance.

Figure 2 shows that δ_1/τ_c is, to a fair approximation, independent of the value of δ_1, and in an earlier experiment Willmarth (1959) obtained for the convection velocity a value of δ/τ_c which was 0·82 of the free-stream velocity.

Alternatively, one can keep a fixed time delay τ_1 and vary the space separation, hence defining another convection velocity δ_c/τ_1, where δ_c satisfies

$$\partial R(\partial, \tau_1)/\partial\delta = 0. \tag{2.4}$$

Figure 2 shows that this velocity, because of the spread of power over the (δ, τ)-plane, cannot be equal to that obtained with constant space separation, even if it is independent of the value of τ_1. The definition of equation (2.4) has since been adopted by most workers, including Willmarth & Wooldridge (1962), in view of the greater importance in most problems, and particularly in aerodynamic noise, of the optimum time scale, rather than the optimum space scale (cf. Lighthill 1954).

In § 3, experiments are described in which the convection velocity so defined for the axial component of velocity was measured at a single value of the time delay τ. The measurements were made in the shear layer of a circular jet at a station two diameters downstream from the nozzle, at various radial positions. The single value of τ was chosen such that the maximum correlation coefficient at this value of τ was close to 0·5. Such an approach readily gives an idea of the way in which convection velocity varies from point to point across a shear layer but, since the velocity so defined is strictly a function of τ, the significance of the velocity obtained at one value of τ is uncertain. Earlier experiments (Ffowcs Williams 1960) had, however, suggested that in this experimental situation the convection velocity is independent of τ, at least for large values of τ.

Subsequently it was desired to compare the results of §3 with the convection velocity for the transverse component of velocity fluctuations in the jet, and the opportunity was taken to perform the measurements over a range of values of τ. The experiments are described in §4, and it will be seen (figure 7) that the convection velocity at a fixed point in the flow can show variations of the order of 30 % over the range of time delay covered in the experiment.

This suggests that, if a single convection velocity is to be meaningful, a more exacting definition is needed, and in a private communication Ffowcs Williams pointed out that a suitable choice was the velocity satisfying the condition

$$\frac{\partial}{\partial U}\int_{-\infty}^{\infty} R(U\tau,\tau)\,d\tau = 0. \tag{2.5}$$

(Note that the previous condition may be written $\partial\{R(U\tau,\tau)\}/\partial U = 0$.) The integral time scale is a maximum in the frame of reference moving downstream with this velocity. An analogous definition has been used by Phillips (1957) to obtain the velocity of the reference point in which the integral time scale of a particular wave-number $\mathbf{k}$ is a maximum. We shall discuss this velocity, say $U_c(\mathbf{k})$, at a later stage in the paper.

Although the velocity defined by equation (2.5) is relevant to the study of aerodynamic noise, it seems to have little significance as a convection velocity of fluid particles. Interestingly enough, though, it can be identified with an average of the velocities of eddies of all wave-numbers, a point which is readily demonstrated from a study of the relevant power spectral functions. Suppose that a stationary, homogeneous turbulent field along a line parallel to the mean flow is made up of many waves with one-dimensional wave-number k, moving with a range of velocities U. The frequency ω generated at a fixed point by a particular wave as it moves downstream will be equal to $-kU$. The turbulent field at the point has a cross-power spectrum $M(k,\omega)$ which is the double Fourier transform of the covariance $R(\delta,\tau)$ (see table 1). We now define the related power spectral function

$$W(k,U) = M(k,-kU), \tag{2.6}$$

and show this to be a suitable basis for the definition of a convection velocity which is dependent on eddy size or, more strictly, is a function of the wave-number component k in the mean-flow direction. Just as the conventional energy spectrum $M(k,\omega)$ represents the amplitude of an elementary perturbation, harmonic in space and time, so the modified spectral function $W(k,U)$ represents the amplitude of an elementary perturbation harmonic in space and moving with a velocity U, $W(k,U)\,e^{ik(x-Ut)}$. The real stationary, homogeneous fluctuating field may be regarded as a superposition of an infinite set of either of these elementary systems, but the second approach based on $W(k,U)$ is particularly suitable for the analysis of convected phenomena, since the real field can usefully be regarded as a superposition of many uniformly convected non-dispersive wave systems. For each element the velocity U has a well-defined meaning and is equal to both the phase and group velocities of the elementary wave.

We can define a dominant convection velocity at a particular wave-number, $U_c(k)$, as the velocity that makes the spectral function $W(k, U)$ a maximum, i.e.

$$\{\partial W(k, U)/\partial U\}_{U=U_c(k)} = 0. \tag{2.7}$$

This definition of $U_c(k)$ is identical with that used by Phillips (1957) in his study of water waves generated by a turbulent wind, but is here restricted to one space dimension. Phillips defined his convection velocity to be that of a moving frame of reference in which the integral time scale $\int_{-\infty}^{\infty} F(\mathbf{k}, \tau)_m \, d\tau$ was greatest (see table 1), which in one dimension is also the condition for the maximum of $W(k, U)$.

Spectrum	Relevant transform
$M(k, \omega)$	$(2\pi)^{-2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R(\delta, \tau)\exp\{i(k\delta+\omega\tau)\}\,d\delta\,d\tau$
$W(k, U)$	$(2\pi)^{-2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R(\delta, \tau)\exp\{ik(\delta-U\tau)\}\,d\delta\,d\tau$
$J(k, U)$	$k(2\pi)^{-2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R(\delta, \tau)\exp\{ik(\delta-U\tau)\}\,d\delta\,d\tau$
$R_\omega(\delta)$	$(2\pi)^{-1}\int_{-\infty}^{\infty} R(\delta, \tau)\exp(i\omega\tau)\,d\tau$
$F(\mathbf{k}, \tau)_m$	$\exp(i\mathbf{k}\,.\,\mathbf{U}\tau)(2\pi)^{-1}\int_{-\infty}^{\infty} R(\boldsymbol{\delta}, \tau)\exp(i\mathbf{k}\,.\,\boldsymbol{\delta})\,d\boldsymbol{\delta}$

TABLE 1. Table of spectra.

Now, just as the frequency spectrum $M(\omega)$ is derived from the more general function $M(k, \omega)$ by the integral

$$M(\omega) = \int_{-\infty}^{\infty} M(k, \omega)\, dk, \tag{2.8}$$

so we can define a spectral function $W(U)$ by

$$W(U) = \int_{-\infty}^{\infty} W(k, U)\, dk. \tag{2.9}$$

Furthermore, just as the peak value of $M(\omega)$ occurs at a characteristic frequency, so the velocity at which the spectrum $W(U)$ peaks will be an important characteristic velocity. We have already defined the velocity $U(k)$ as the velocity of maximum energy at a particular wave-number (equation (2.7)), so we now define the overall convection velocity U_c as the value at which the peak of the integrated spectrum $W(U)$ peaks, i.e.

$$\{\partial W(U)/\partial U\}_{U=U_c} = 0. \tag{2.10}$$

But on making use of the identity

$$\int_{-\infty}^{\infty} W(k, U)\, dk = (2\pi)^{-1} \int_{-\infty}^{\infty} R(U\tau, \tau)\, d\tau, \tag{2.11}$$

we see that our definition of the overall convection velocity U_c is the same as that based on the maximum integral time scale of $R(\delta, \tau)$, equation (2.5). Thus this treatment of convection velocity through properties of wave-number spectra provides a unified approach of which the particular definitions adopted by different workers are special cases.

The property of greatest interest in the present study is the wave-number/velocity spectral density $W(k, U)$, and this can be obtained experimentally from a double Fourier transform of the covariance $R(\delta, \tau)$. Since $R(\delta, \tau)$ can be measured easily with modern equipment, this is an obvious method of obtaining the spectrum $W(k, U)$. An alternative method, having several experimental advantages, is to measure the spatial covariance with zero time delay, at a frequency ω, denoted by $R_\omega(\delta)$. This function is the real part of the single Fourier transform of $R(\delta, \tau)$ with respect to τ, and is thus necessarily an even function of ω. If we now transform $R_\omega(\delta)$ with respect to δ, we obtain the (real) sum

$$M(k, \omega) + M(k, -\omega),$$

so that

$$W(k, \omega/k) + W(k, -\omega/k) = 2(2\pi)^{-1} \int_0^{\infty} R_\omega(\delta) \cos k\delta\, d\delta. \tag{2.12}$$

The method thus provides a measure of the total power at positive and negative frequencies, or rather at positive and negative velocities, since in homogeneous, stationary flow $M(k, \omega) = M(-k, -\omega)$. The simple measurement of $R_\omega(\delta)$ with narrow band filters does not allow the separation of the positive and negative velocity components, but in flows where the convection is strong there will be little energy at negative velocities, so that

$$W(k, -\omega/k) \approx 2(2\pi)^{-1} \int_0^{\infty} R_\omega(\delta) \cos k\delta\, d\delta. \tag{2.13}$$

The effect of this approximation is that there will exist ranges of wave-number and velocity where the accuracy of $W(k, U)$ cannot be guaranteed, and in these regions the true spectrum could be obtained by various methods, perhaps the simplest of which involves measuring $R_{i\omega}(\delta)$, the spatial covariance of filtered signals with a relative phase shift of $\frac{1}{2}\pi$ at frequency ω. It is then easily shown that

$$W(k, -\omega/k) = 2(2\pi)^{-1} \int_0^{\infty} \{R_\omega(\delta) \cos k\delta + R_{i\omega}(\delta) \sin k\delta\}\, d\delta. \tag{2.14}$$

However, to obtain the velocity $U_c(k)$ one needs measurements of $W(k, U)$ only in the vicinity of its maximum, so that generally the approximate relation (2.13) can be used without serious error.

Although strictly only one signal need be filtered to obtain $R_\omega(\delta)$, in practice it is essential to filter both with identical filters, because of the inevitable large phase shifts through the pass band of the filter. The use of two filters also improves the rejection of frequencies outside the pass band and limits the signal amplitude fed to the multiplier, allowing greater accuracy and an improved signal-to-noise ratio. Experimentally, the elimination of the usual tape- or drum-time-delay unit has a considerable advantage, since the overall signal-to-noise ratio and frequency response of the measurements will generally be limited by this part of the equipment.

An experiment in which the spectrum $W(k, U)$ was measured by the filtered space-correlation technique is described in § 6, for one position in the shear layer of a circular jet.

3. Experimental equipment

The experiments were performed in a circular jet of 2 in. diameter with an exit velocity of 330 ft./sec. The hot wires were of 0·0002 in. diameter and about 0·02 in. length, and were made by etching away the centre length of the copper coating from a copper-plated tungsten wire soldered to the ends of the hot-wire holder. The technique is described in greater detail by Bradshaw & Johnson (1963). These holders were mounted on a traversing gear that allowed one wire to be fixed at any point in the flow, and the other to be traversed in a direction parallel to the jet axis by a motor drive. The wires were operated at constant temperature with a resistance ratio $R/Ra = 2$, corresponding to a wire temperature of about 250 °C, by Disa model 55 A 01 hot-wire anemometers, and the output signals from the anemometers were fed to single-valve linearizers having a response adjusted to match the hot-wire non-linear response and give a voltage signal directly proportional to velocity. The linearizers in turn fed a four-head time-delay tape recorder (Data Recording model 480/100 using F.M. recording), and the output signals, now with a relative time delay, could be passed to a time-division multiplier and then integrated for a fixed time to give a signal proportional to $R(\delta, \tau)$. The linearizers, integrator, timer and multiplier are described in detail by Bradshaw & Johnson (1963).

For the measurements of transverse velocity fluctuations, crossed-wire probes were used, requiring the use of two more Disa anemometers and linearizers, and two difference amplifiers. For the measurements described in § 6 the signals from two linearized anemometers were fed into two Brüel and Kjaer model 2111 audio-frequency spectrometers, giving filtered signals with a band-width of $\frac{1}{3}$-octave which were then multiplied and integrated as in the earlier experiment. The Fourier transform to give the spectrum $W(k, U)$ was evaluated using the Deuce digital computer at the NPL.

4. Measurements of convection velocity of axial components

The fixed wire was positioned two diameters downstream of the jet nozzle and half a diameter from the jet axis, roughly in the centre of the shear layer. The moving wire could be traversed in the downstream direction (see figure 3). The signal from the moving wire was delayed in time by an amount $0{\cdot}537D/U_{\max}$, and the covariance of the axial velocity component was measured for a range of wire separation δ that covered the covariance maximum. The convection velocity $U_{c\tau}$ was obtained as the value of the ratio δ/τ that maximized $R(\delta, \tau)$ at the selected

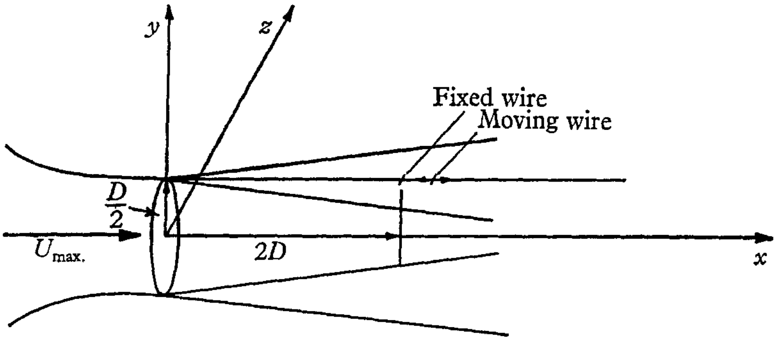

FIGURE 3. Schematic jet arrangement.

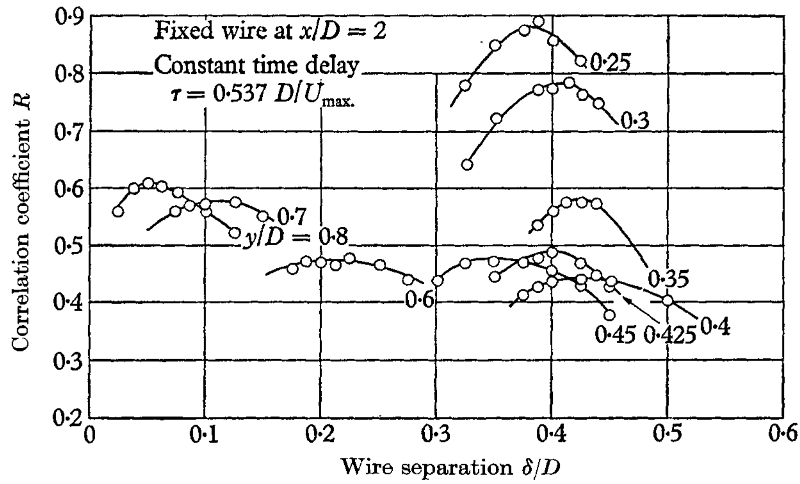

FIGURE 4. Longitudinal velocity space–time correlation.

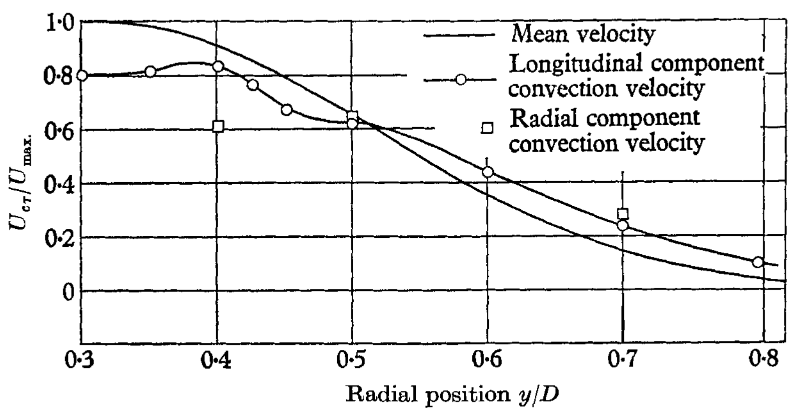

FIGURE 5. Variation of convection velocity across shear layer at $x/D = 2$.

value of τ, which was chosen to give a peak correlation coefficient of about 0·5. The experiment was repeated for a range of radial positions to give the results shown in figure 4, and the distribution of convection velocity across the shear

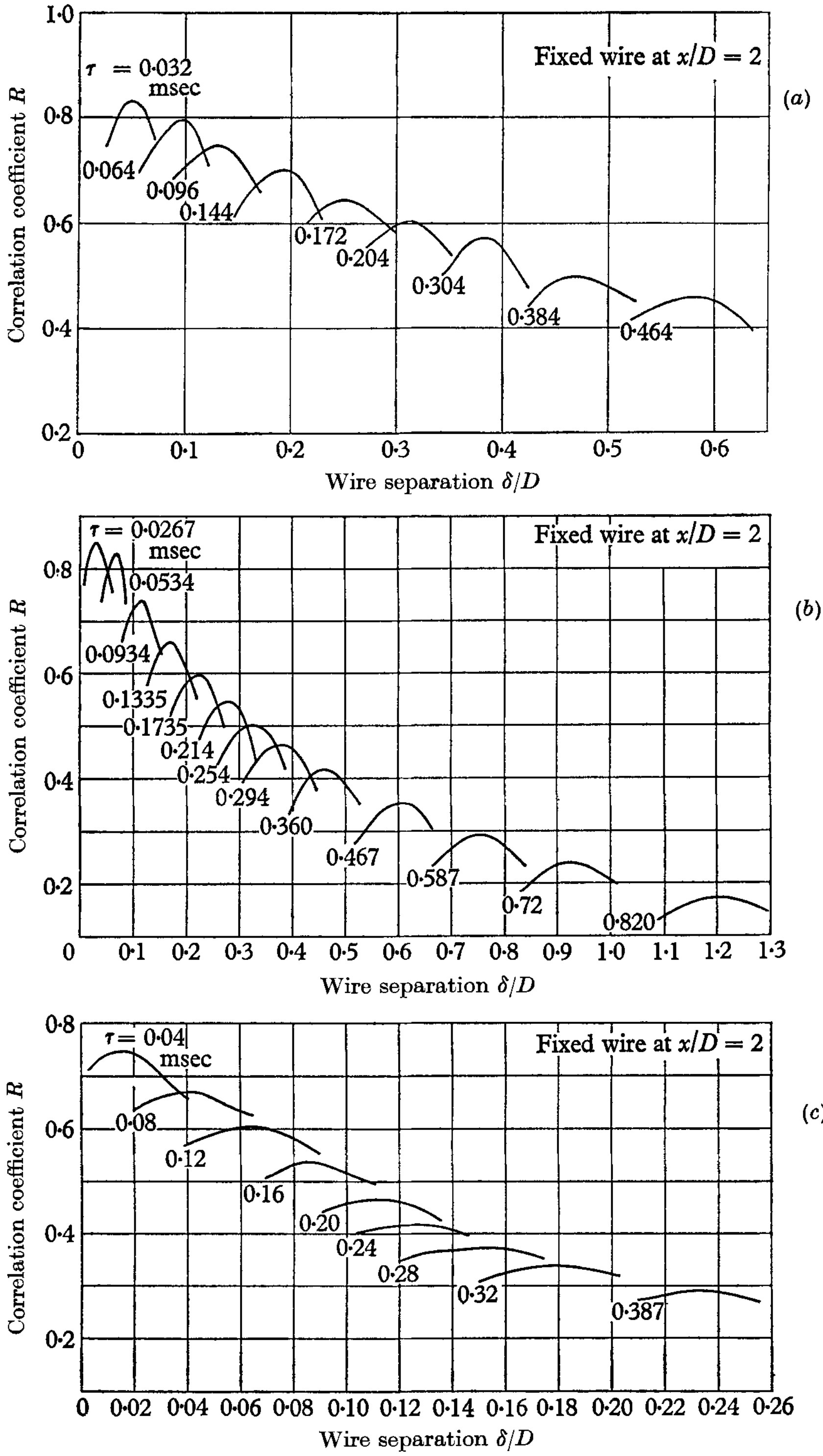

FIGURE 6. Radial velocity space–time correlation at (a) $y/D = 0{\cdot}4$, (b) $y/D = 0{\cdot}5$, and (c) $y/D = 0{\cdot}7$.

layer is shown in figure 5. The results are substantially the same as those obtained by Davies, Fisher & Barratt (1963). The convection velocity defined on this basis appears to change much less across the shear layer than does the mean velocity, which is shown for comparison on the same figure, and this suggests that in the lower-intensity regions at the extremities of the curve one is observing in part the field of the turbulence in the high-intensity region, travelling at about the mean velocity in that region.

5. Measurements of convection velocity of radial component

An experiment similar to that of §4 was performed, using crossed-wire probes with wires at approximately $\pm 45°$ to the jet axis and lying in the (x, y)-plane two diameters downstream of the jet nozzle. The two radial-component signals

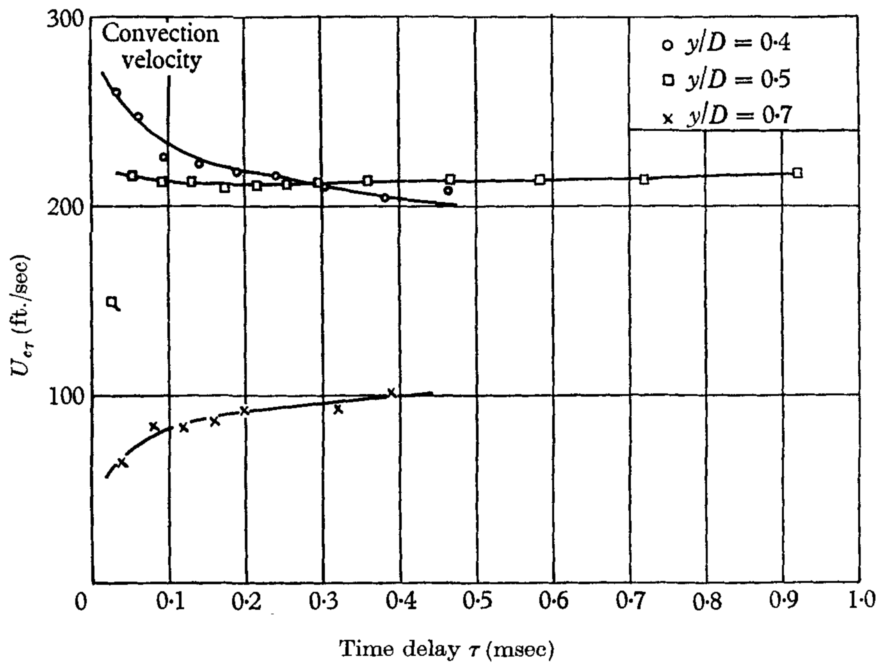

FIGURE 7. Radial component convection velocity.

were processed to give the convection velocity, defined as in §4 as the value of the ratio δ/τ that maximized $R(\delta, \tau)$, but in this case measurements were made over a range of values of τ, and for only three values of y/D, 0·4, 0·5 and 0·7. The results are shown in figure 6, and the variation of convection velocity with time delay and radial position is shown in figure 7. An average convection velocity, given by the slope of the best straight line through the optimum values of δ and τ, is also shown in figure 5 for the three positions where it was measured, and exhibits even less variation with radial position than does the axial-component convection velocity. Recent measurements by Bradshaw, Ferriss & Johnson (1963) have indicated that the radial-component lateral scale is roughly twice that of the axial-component, so that a larger region of approximately constant convection velocity might be expected if the observed velocity perturbations are in part the induced field of powerful eddies in the centre of the shear flow.

Perhaps the more important result of this experiment, however, is that the local convection velocity is not independent of time delay τ, particularly at the extremities of the shear layer, and this result suggested the experiment of §6.

6. Measurements of $U_c(k)$

Two axial-component wires were again operated at a resistance ratio $R/Ra = 2$ by two Disa anemometers with linearizers. The outputs were filtered by two Brüel and Kjaer audio spectrometers whose outputs were fed to the multiplier and integrator. Both filters were initially set to a centre frequency of 400 c/s, with a bandwidth of $\frac{1}{3}$-octave. As before, the fixed wire was positioned two diameters downstream of the jet nozzle and half a diameter from the jet axis, and the moving wire was traversed upstream and downstream of this position to cover the range of significant correlation. In the case of the 400 c/s band, this range extended from the jet nozzle to a point eight diameters downstream. The experiment was repeated at centre frequencies of 800, 1600, 3150, 6300 and 12,500 c/s. A typical correlation curve is shown in figure 8. For experimental convenience values of the correlation coefficient $R_\omega(\delta)/(\overline{u^2(x)}\ \overline{u^2(x+\delta)})^{\frac{1}{2}}$ were measured, and the failure of this quantity to reach unity for zero separation in

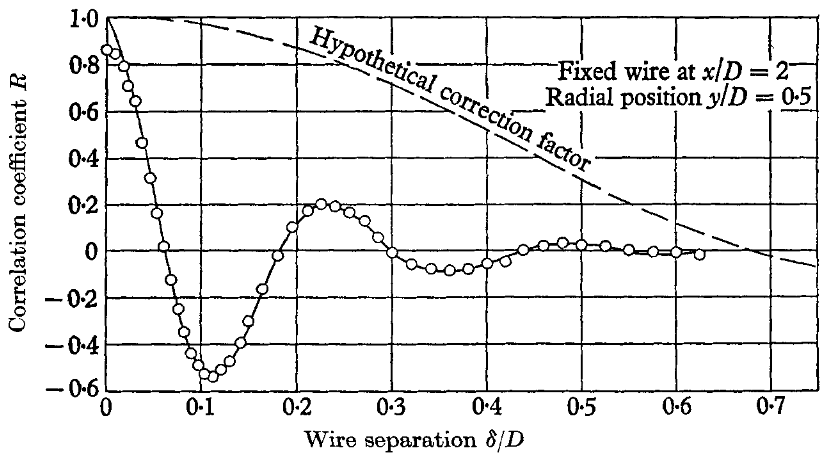

FIGURE 8. Longitudinal velocity filtered space correlation at 6300 c/s.

figure 8 arises from the necessity of displacing the two wires slightly in the tangential direction to avoid wake interference by the upstream wire. This difficulty does not arise at lower frequency where the effective lateral scale is greater. Curves of $R_\omega(\delta)$ *vs* δ were constructed, and ordinates from these curves were used in a standard programme to compute the Fourier cosine transform. Figure 9 is the computed transform of the curve shown in figure 8, and is typical of these used to generate the spectrum $W(k, U)$ shown in figure 10. Since the velocity U is defined as the ratio $-\omega/k$, the constant-frequency lines on which the experimental points lie appear as hyperbolae. Also shown in figure 10 are the

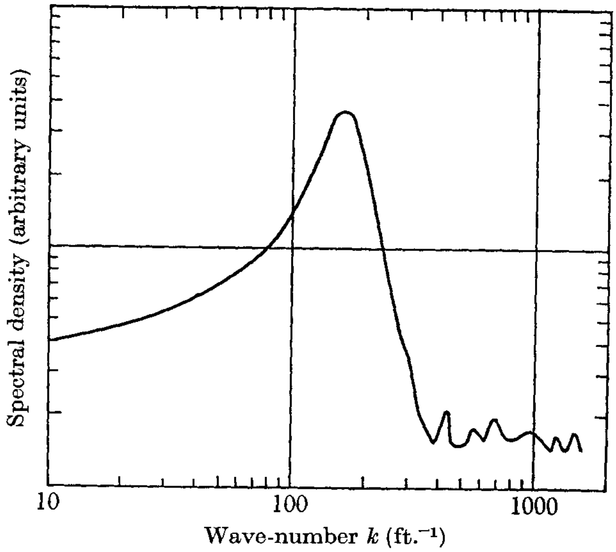

FIGURE 9. Fourier (cosine) transform of figure 8.

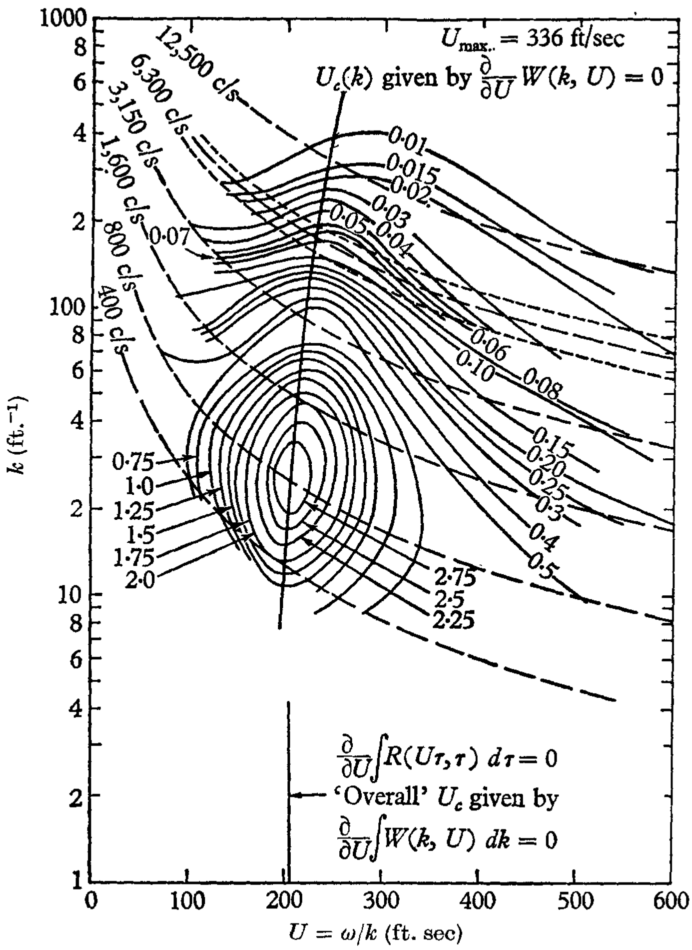

FIGURE 10. The spectrum $W(k, U)$ at $x/D = 2$, $y/D = 0{\cdot}5$.

bandwith limits of the 6300 c/s filter, and this can be used to estimate the inaccuracy of the results arising from the finite bandwith of the filters. The usual correction to filtered space correlation measurements involves invoking Taylor's hypothesis to convert the filtered autocorrelation function into the corresponding space correlation, and this correction factor is shown in figure 8. However, it is easily shown (Wills 1963) that the true effect of an ideal slot filter is to give a mean reading of the energy over the filter bandwith at the particular wave-number concerned; i.e. the measured spectrum is given by

$$W^m(k, -\omega/k) = \int_{\omega-\Delta\omega}^{\omega+\Delta\omega} W(k, -\omega/k)\, d\omega. \tag{6.1}$$

The main inaccuracy in the measured spectrum will occur at high velocity where the constant-energy lines become almost parallel to constant-frequency lines, but in the high-intensity region the effect of the filter is restricted to a levelling-off of the spectrum peak, and since the pattern is roughly symmetrical either side of the peak there will be little error in the value of $U_c(k)$ obtained.

As the spectrum $W(k, U)$ is not a familiar one, it is perhaps worth illustrating how it is related to spectra according to more usual definitions. Figure 11 shows two constant-energy contours of the cross-power spectrum $M(k, \omega)$ in convected

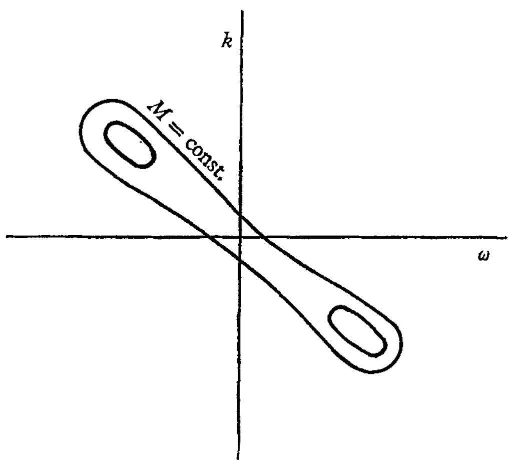

FIGURE 11. The spectrum $M(k, w)$.

turbulence, and the main energy is concentrated around a convection-velocity line given by the ratio $-\omega/k$. The wave-number/velocity spectrum $J(k, U)$ suggested by Ffowcs Williams (1961) to be relevant to the study of convected turbulence is related to this by the formula (see table 1)

$$J(k, U) = kM(k, -kU). \tag{6.2}$$

The spectrum depicted by the contours of figure 11 would transform to give the constant energy contours of $J(k, U)$ shown in figure 12, and the main energy is again seen to be concentrated around a particular value of velocity. Both the spectra $M(k, \omega)$ and $J(k, U)$ can be integrated over their respective spaces to give

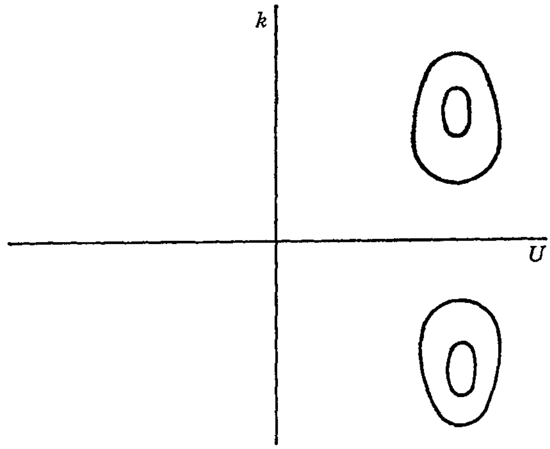

FIGURE 12. The spectrum $J(k, U)$.

the turbulent intensity $\overline{u^2}$, but the spectrum $J(k, U)$ appears to have less significance in the convection-velocity study than the closely related function

$$W(k, U) = k^{-1}J(k, U)$$

which is the subject of the present investigation, and which lends itself readily to the definition of the convection velocity $U_c(k)$ and the overall convection velocity U_c.

The contours of figure 11 are re-plotted in figure 13 to give contours of the spectrum $W(k, U)$, and it can be seen that even at very high velocities there will be appreciable energy at small wave-numbers. The turbulent field has been regarded as an assemblage of undamped harmonic waves, which it is well known (see, for instance, Phillips 1960) can radiate no sound unless the wave velocity is

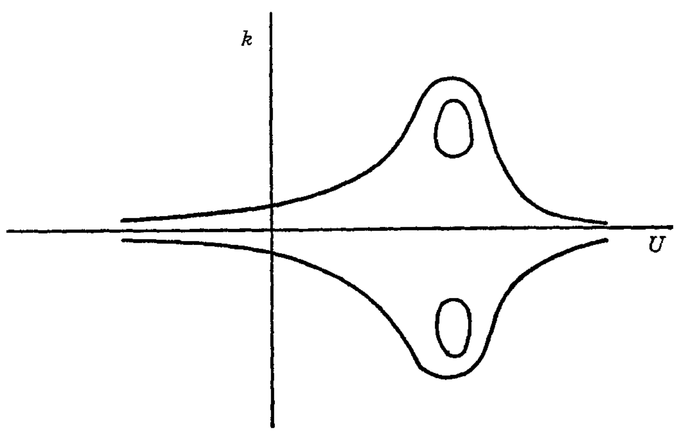

FIGURE 13. The spectrum $W(k, U)$.

supersonic with respect to the external field. Thus the energy in the supersonic region of figure 13 is the only part that contributes to the sound radiation from the flow, and a typical wave-number in this region is much smaller than that in the region of maximum intensity.

The experimental results of figure 10 show that maximum intensity to occur at a wave-number of about 25 ft.$^{-1}$, corresponding to a wavelength in the axial direction of 3 in. The convection velocity $U_c(k)$ at this wave-number is 205 ft./sec, or 0·62 times the jet exit velocity. This velocity is also approximately equal to the overall convection velocity U_c, and slightly less than the mean velocity at this radial position. Even at the peak-energy wave-number, only half the energy lies within the velocity range 150–250 ft./sec, so that the assumption sometimes made that wave-number and frequency are related by a single velocity is unlikely to give useful results, except perhaps when used to indicate orders of magnitude. At wave-numbers remote from the peak-energy position, the spread of energy over velocity is even greater.

The value of $U_c(k)$ away from the peak-energy position shows a slight increase with increasing wave-number, but the experimental difficulties of obtaining accurate results at high wave-number precludes the possibility of giving an accurate empirical value to $U_c(k)$ at present. It is not yet clear whether the variation is a dynamical effect of the local motion, or the impressed field of faster-moving eddies nearer the jet axis.

7. Conclusions

The experiments of §4 and those of several other workers (e.g. Willmarth & Wooldridge 1962) indicate that, in some turbulent shear flows of practical interest, the usual simple definitions of a single convection velocity of the fluctuations lead to an ambiguous result. It is evident that we must either adopt a more refined definition of a single velocity, or allow the convection velocity to be a function of at least one parameter. If we choose the latter course, the possible variables are the space or time separation, or the transformed variables wave-number or frequency. From the point of view of problems where the turbulence acts as a forcing function for some motion, as in water-wave generation or aerodynamic noise production, the obvious choice of variables is wave-number because of the identification of each wave-number with a particular velocity. The approach described here leads naturally from the distribution of turbulent energy over wave-number and velocity to the convection velocity $U_c(k)$, and thence to an integral convection velocity U_c that eliminates the ambiguity of the usual definitions.

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